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THE EFFECT OF SUBCOOLED LIQUID ON FILM BOILING ABOUT A VERTICAL HEATED SURFACE IN A POROUS MEDIUM

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Abstract — The problem of steady natural convection film boiling about a heated isothermal vertical plate in a porous medium filled with a subcooled liquid is considered. With the boundary layer approximations, similarity solutions are obtained for the buoyancy-induced flow in the vapor and subcooled liquid layers with a distinct interface. At a given vapor Rayleigh number, the Nusselt number is found to be uniquely dependent on the vapor film's dimensionless thickness, which in turn depends on three dimensionless parameters related to the degree of superheating of the wall, the extent of the subcooling of the surrounding liquid, and a property ratio of the vapor and the liquid phases. It is found that the effect of the increase of the subcooling of the surrounding fluid tends to decrease the vapor boundary layer thickness, increase the liquid boundary layer thickness and increase the surface heat flux. On the other hand, the increase of the wall superheating tends to increase the vapor layer thickness, decrease the liquid layer thickness and increase the surface heat flux. Application to boiling heat transfer about a dike intruded into an aquifer is discussed.

NOMENCLATURE

- c, specific heat of the convective fluid;
- C_L, a quantity defined in equation (38);
- f, dimensionless stream function;
- h, local heat transfer coefficient;
- h_{fg}, latent heat of vaporization;
- K, permeability of the porous medium;
- k, thermal conductivity of the porous medium;
- $\dot{m}$, mass flux;
- Nu_x, local Nusselt number;
- p, pressure;
- q, local heat transfer rate;
- R, $\equiv \dfrac{\rho_v}{\rho_\infty}\left[\dfrac{\mu_L \alpha_v(\rho_\infty - \rho_v)C_{pL}}{\mu_v \alpha_L \rho_\infty \beta_L h_{fg}}\right]^{1/2}$, property ratio of the vapor and the liquid phase;
- Ra_x, local Rayleigh number;
- Sc, $\equiv c_{pL}(T_s - T_\infty)/h_{fg}$, dimensionless degree of subcooling of liquid;
- Sh, $\equiv c_{pv}(T_w - T_s)/h_{fg}$, dimensionless degree of wall superheating;
- T, temperature;
- u, Darcy's velocity in x-direction;
- v, Darcy's velocity in y-direction;
- x, coordinate along the surface;
- y, coordinate perpendicular to the surface.

Greek symbols

- α, equivalent thermal diffusivity;
- β, the coefficient of thermal expansion;
- δ, boundary layer thickness;
- η, similarity variable;
- θ, dimensionless temperature;
- μ, viscosity of the convective fluid;
- ρ, density of the convective fluid;
- ψ, stream function.

Subscripts

- s, saturated condition;
- v, vapor phase;
- L, liquid phase;
- ∞, condition at infinity;
- w, condition at the wall.

INTRODUCTION

THE PROBLEM of boiling heat transfer in a porous medium has important applications in engineering and geophysics. In a recent paper, Parmentier [1] studied the problem of boiling heat transfer about a heated vertical surface in a permeable medium filled with a subcooled water, with application to dike intrusion in an aquifer. Parmentier [1] postulated that when boiling occurs adjacent to a vertical surface in a porous medium, a thin vapor film will form. With the aid of a p–T phase diagram, he argued that the vapor film and the liquid water are separated by a distinct interface with no mixed region in between. As a result of this approximation, the mathematical formulation of the problem is considerably simplified. By assuming: (a) that the density of the subcooled water is constant; (b) the density of the vapor is small compared with the saturated water; and (c) that heat conduction in the longitudinal direction is small compared with the transverse direction, Parmentier obtained an approximate solution for Nusselt number for this problem.

The assumption that the vapor and liquid phases are separated by a distinct interface has always been made in the classical film boiling heat transfer literature. Earlier work on natural convection film boiling about a vertical plate in a Newtonian fluid has been studied by Bromley [2] and by Ellison [3] based on the assumptions that inertia force of the vapor film is small, the interface velocity is zero, and the temperature profile in the vapor film is linear. Koh [4] studied the same problem based on boundary layer theory, and taking into consideration the momentum transfer at the vapor liquid interface. At the same time, Sparrow and Cess [5] studied the effect of subcooled liquid on film boiling with the assumption of zero interface velocity. Sparrow and Cess's problem was later studied by Nishikawa, Ito and Matsumoto [6] who used the same hydrodynamic interfacial boundary conditions as that of Koh [4].

Most recently, Cheng [7] obtained a similarity solution for stable film boiling about an inclined surface in a porous medium filled with saturated liquid (i.e. zero subcooling), based on the usual assumptions made in the classical film boiling literature. In the present work, the effect of subcooled liquid on stable film boiling about a vertical plate in a porous medium will be considered. The assumptions made in this paper are similar to the previous work [7], and similarity solutions are obtained for both the vapor and liquid phases. These two solutions are interconnected through the interface boundary conditions. Numerical solutions for the similarity equations follow closely the work by Sparrow and Cess [5]. A closed form solution for Nusselt number is obtained in terms of Rayleigh number and the dimensionless boundary layer thickness of the vapor layer; the latter is found to depend on three dimensionless parameters relating to the degree of the superheating of the wall, the extent of the subcooling of the surrounding liquid, and a property ratio of the vapor and the liquid phases. It is found that the effect of the increase of the subcooling of the surrounding fluid tends to decrease both the vapor and the liquid boundary layer thicknesses, and to increase the surface heat flux. On the other hand, the increase of the wall superheating tends to increase the vapor layer thickness, decrease the liquid layer thickness, and increase the surface heat flux. Application to boiling heat transfer about a dike intruded into an aquifer is discussed.

FORMULATION OF THE PROBLEM

Consider the problem of steady heating of an isothermal vertical plate embedded in a porous medium filled with a subcooled liquid as shown in Fig. 1. When the wall temperature T_w is sufficiently higher than the saturated temperature T_s corresponding to its pressure, a vapor film will form adjacent to the vertical plate. To investigate the two-phase buoyancy-induced flow in a porous medium adjacent to a vertical plate, the following assumptions will now be made:

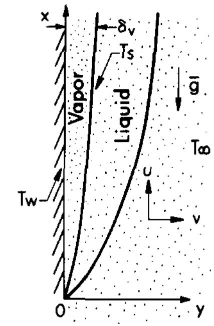

FIG. 1. Coordinate system.

(1) A distinct boundary exists between the vapor and the subcooled liquid with no mixed region in between.

(2) The interface at $y = \delta_v$ is smooth and stable, and is at a constant temperature T_s.

(3) Boundary layer approximations are applicable.

(4) Boussinesq approximations are invoked in the liquid phase so that density is assumed to be constant except in the buoyancy force term where density is assumed to be linearly proportional to the temperature.

(5) All other properties of the liquid and vapor phases and the porous medium are constant. In particular, the density of the vapor is assumed to be constant in the buoyancy force term $(\rho_v - \rho_\infty)g$. This is due to the fact that $\rho_\infty \gg \rho_v$ so that a more accurate representation of the vapor density would not significantly affect the results.

(6) Darcy's law is applicable to both phases.

It is worth noting that assumptions (1)–(5) are the usual approximations used in treating classical film boiling heat transfer problems, and that assumption (4) was not used by Parmentier. With assumptions (1)–(6), the governing equations for the porous medium filled with the superheated vapor at $y < \delta_v$ are

$$\frac{\partial u_v}{\partial x} + \frac{\partial v_v}{\partial y} = 0 \tag{1}$$

$$u_v = -\frac{K}{\mu_v}(\rho_v - \rho_\infty)g \tag{2}$$

$$u_v \frac{\partial T_v}{\partial x} + v_v \frac{\partial T_v}{\partial y} = \alpha_v \frac{\partial^2 T_v}{\partial y^2} \tag{3}$$

while those for the porous medium filled with the subcooled liquid at $y > \delta_v$ are

$$\frac{\partial u_L}{\partial x} + \frac{\partial v_L}{\partial y} = 0 \tag{4}$$

$$u_L = \frac{K\beta_{L\infty}\rho_\infty(T_L - T_\infty)g}{\mu_L} \tag{5}$$

$$u_L \frac{\partial T_L}{\partial x} + v_L \frac{\partial T_L}{\partial y} = \alpha_L \frac{\partial^2 T_L}{\partial y^2} \tag{6}$$

where the subscripts v, L and ∞ denote the quantities associated with the vapor layer, liquid layer and condition at a great distance from the heating plate; u and v are the Darcy's velocities in the x and y directions; ρ, μ and β are the density, viscosity and thermal expansion coefficient of the convecting fluid; K and α are the permeability and the equivalent thermal diffusivity of the porous medium; p and T are the pressure and temperature. It is worth noting that equation (2) indicates that the vertical velocity in the vapor layer is constant.

The boundary conditions at the wall and at a great distance from the wall are

$$y = 0, \quad v_v = 0, \quad T_v = T_w, \tag{7a, b}$$

$$y \to \infty, \quad u_L = 0, \quad T_L = T_\infty \tag{8a, b}$$

where $T_w > T_s \geq T_\infty$.

At the vapor–liquid interface at $y = \delta_v$, the continuity of temperature demands

$$y = \delta_v, \quad T_v = T_s = T_L. \tag{9}$$

From the continuity of mass flow across the interface, we have

$$\rho_v\left(v_v - u_v\frac{\mathrm{d}\delta_v}{\mathrm{d}x}\right)_{y=\delta_v} = \rho_L\left(v_L - u_L\frac{\mathrm{d}\delta_L}{\mathrm{d}x}\right)_{y=\delta_v} = \dot{m}_\delta \tag{10}$$

where $\dot{m}_\delta$ is the mass flux through the interface.

The energy balance across the interface gives

$$-k_{m,v}\left(\frac{\partial T_v}{\partial y}\right)_{y=\delta_v} = \dot{m}_\delta h_{fg} - k_{m,L}\left(\frac{\partial T_L}{\partial y}\right)_{y=\delta_v} \tag{11}$$

where k is the equivalent thermal conductivity of the porous medium and h_{fg} is the latent heat of vaporization of the liquid at T_s. Equation (11) shows that the energy across the interface is partly conducted into the subcooled liquid and partly is used to evaporate liquid at a rate of $\dot{m}_\delta$.

We now introduce the stream functions for the liquid and vapor phases such that

$$u_v = \frac{\partial \psi_v}{\partial y}, \quad v_v = -\frac{\partial \psi_v}{\partial x} \tag{12a, b}$$

$$u_L = \frac{\partial \psi_L}{\partial y}, \quad v_L = -\frac{\partial \psi_L}{\partial x} \tag{13a, b}$$

so that the continuity equations for both phases are satisfied automatically. In terms of the stream functions, equations (2), (3), (5) and (6) become

$$\frac{\partial \psi_v}{\partial y} = \frac{K}{\mu_v}(\rho_\infty - \rho_v)g \tag{14}$$

$$\frac{\partial \psi_v}{\partial y}\frac{\partial T_v}{\partial x} - \frac{\partial \psi_v}{\partial x}\frac{\partial T_v}{\partial y} = \alpha_v\frac{\partial^2 T_v}{\partial y^2} \tag{15}$$

and

$$\frac{\partial \psi_L}{\partial y} = \frac{K}{\mu_L}\rho_\infty \beta_{L\infty} g(T_L - T_\infty) \tag{16}$$

$$\frac{\partial \psi_L}{\partial y}\frac{\partial T_L}{\partial x} - \frac{\partial \psi_L}{\partial x}\frac{\partial T_L}{\partial y} = \alpha_L\frac{\partial^2 T_L}{\partial y^2}. \tag{17}$$

Boundary conditions (7a), (8a), (10) and (11) in terms of stream functions are

$$y = 0, \quad \frac{\partial \psi_v}{\partial x} = 0 \tag{18}$$

$$y \to \infty, \quad \frac{\partial \psi_L}{\partial y} = 0 \tag{19}$$

$$\rho_v\left[\frac{\partial \psi_v}{\partial x} + \frac{\partial \psi_v}{\partial y}\frac{\mathrm{d}\delta_v}{\mathrm{d}x}\right]_{y=\delta_v} = \rho_L\left[\frac{\partial \psi_L}{\partial x} + \frac{\partial \psi_L}{\partial y}\frac{\mathrm{d}\delta_L}{\mathrm{d}x}\right]_{y=\delta_v} \tag{20}$$

$$k_{m,v}\left(\frac{\partial T_v}{\partial y}\right)_{y=\delta_v} = -h_{fg}\rho_v\left[\frac{\partial \psi_v}{\partial x} + \frac{\partial \psi_v}{\partial y}\frac{\mathrm{d}\delta}{\mathrm{d}x}\right]_{y=\delta_v} + k_{m,L}\left(\frac{\partial T_L}{\partial y}\right)_{y=\delta_v}. \tag{21}$$

Equations (14)–(17) with boundary conditions (7b), (8b), (9), (18)–(21) will now be solved by similarity transformations. To this end, we first introduce the following new dependent and independent variables for the vapor layer

$$\eta_v = \sqrt{(Ra_{x,v})}\,y/x \tag{22a}$$

$$\psi_v = \alpha_v\sqrt{(Ra_{x,v})}\,f_v(\eta) \tag{22b}$$

$$\theta_v(\eta_v) = \frac{T_v - T_s}{T_w - T_s} \tag{22c}$$

where $Ra_{x,v} = K(\rho_\infty - \rho_v)gx/\mu_v\alpha_v$ is the local Rayleigh number for the vapor phase. In terms of these variables, the governing equations (14) and (15) with boundary conditions (7a, b) and (9) are

$$f_v' = 1 \tag{23}$$

$$2\theta_v'' + f_v\theta_v' = 0 \tag{24}$$

with boundary conditions

$$f_v(0) = \theta_v(0) - 1 = 0 \tag{25a, b}$$

$$\theta_v(\eta_{v\delta}) = 0 \tag{26}$$

where the prisms denote the differentiation with respect to η_v, and $\eta_{v\delta}$ is the dimensionless vapor boundary layer thickness i.e.

$$\eta_{v\delta} = (\eta_v)_{y=\delta_v} = \sqrt{(Ra_{x,v})}\,\delta/x.$$

Equations (23) and (24) with boundary conditions (25) and (26) have the following exact solutions

$$f_v = \eta_v \tag{27}$$

$$\theta_v = 1 - \operatorname{erf}\left(\frac{\eta_v}{2}\right)\Big/\operatorname{erf}\left(\frac{\eta_v\delta}{2}\right). \tag{28}$$

It follows from equations (27) and (28) that

$$u_v = \frac{K}{\mu_v}(\rho_\infty - \rho_v)g, \tag{29a, b}$$

$$v_v = \frac{1}{2}\sqrt{\left[\frac{K\alpha_v}{\mu_v x}(\rho_\infty - \rho_v)g\right]}(f_v - \eta_v f'_v)$$

and

$$\theta'_v = -\left[\sqrt{\pi}\exp\left(\frac{\eta_v^2}{4}\right)\operatorname{erf}\left(\frac{\eta_{v\delta}}{2}\right)\right]^{-1}. \quad (30)$$

Thus, the velocity and temperature fields in the vapor phase are uniquely determined if the value of the dimensionless vapor boundary layer thickness, $\eta_{v\delta}$, is known. However, the value of $\eta_{v\delta}$ can only be determined by solving simultaneously the equations for the liquid phase and satisfying interface boundary conditions. To solve the equations for the liquid phase, we introduce the following new variables for the liquid layer

$$\eta_L = \sqrt{(Ra_{x,L})}(y - \delta_v)/x \quad (31a)$$

$$\psi_L = \alpha_L\sqrt{(Ra_{x,L})}f_L(\eta_L) \quad (31b)$$

$$\theta_L(\eta_L) = \frac{T_L - T_\infty}{T_s - T_\infty} \quad (31c)$$

where $Ra_{x,L} = \rho_\infty K\beta_L g(T_s - T_\infty)x/\mu_L\alpha_L$ is the local Rayleigh number for the liquid layer. In terms of the variables given by equations (31), the governing equations for the liquid phase, i.e. equations (16) and (17) become

$$f'_L = \theta_L \quad (32)$$

$$2\theta''_L + f_L\theta'_L = 0. \quad (33)$$

Equation (32) shows that the dimensionless vertical velocity and dimensionless temperature are identical in the liquid layer. The interface boundary conditions (at $y = \delta_v$, i.e. at $\eta_v = \eta_{v\delta}$ for the vapor layer and at $\eta_L = 0$ for the liquid layer) for the continuity of temperature and mass flux are

$$\theta_L(0) = 1 \quad (34)$$

$$f_L(0) = -\frac{\dot{m}_\delta 2\sqrt{x}}{\rho_\infty[\alpha_L K_L\rho_\infty\beta_L g(T_s - T_\infty)/\mu_L]^{1/2}} = \frac{R}{\sqrt{Sc}}\eta_{v\delta} \quad (35)$$

where $Sc = c_{pL}(T_s - T_\infty)/h_{fg}$ is a measure of the degree of subcooling of the liquid and

$$R \equiv \frac{\rho_v}{\rho_\infty}\left[\frac{\mu_L\alpha_v(\rho_\infty - \rho_v)c_{pL}}{\mu_v\alpha_L\rho_\infty\beta_L h_{fg}}\right]^{1/2}$$

with $c_{pL} = k_{m,L}/\rho_\infty\alpha_L$ denoting the specific heat of the liquid. It is worth noting that boundary condition (35) is related to the rate of evaporation.

The boundary conditions at a great distance from the wall are

$$f'_L(\infty) = 0 \quad (36)$$

$$\theta_L(\infty) = 0 \quad (37)$$

It follows from equation (31b) that the velocities in the liquid layer are

$$u_L = \frac{K}{\mu_L}\rho_\infty\beta_L g(T_s - T_\infty)f'_L \quad (38a)$$

and

$$v_L = -\frac{1}{2}\sqrt{\left[\frac{\alpha_L K\rho_\infty\beta_L g(T_s - T_\infty)}{\mu_L x}\right]} \times [f_L - C_L(y/\sqrt{x})f'_L] \quad (38b)$$

where

$$C_L \equiv \left[\frac{K\rho_\infty\beta_L g(T_s - T_\infty)}{\mu_L\alpha_L}\right]^{1/2}.$$

Note that if the values of $\eta_{v\delta}$, R and Sc are prescribed, equations (32) and (33) with boundary conditions (34)–(37) are identical to the problem of single-phase free convection in a porous medium adjacent to a vertical plate with suction which has been solved by Cheng [8].

The coupling of the equations for the vapor and liquid layers is through the dimensionless vapor boundary layer thickness $\eta_{v\delta}$ and the energy balance equation across the interface which is given by

$$Sh = \left[\frac{Sc^{3/2}}{R}\theta'_L(0) - \frac{\eta_{v\delta}}{2}\right] \Big/ \theta'_v(\eta_{v\delta}) \quad (39)$$

where $Sh = c_{pv}(T_w - T_s)/h_{fg}$ is a measure of the wall superheating with $c_{pv} = k_v/\rho_v\alpha_v$. Note that in arriving at equation (39) we have used the relation $\delta_v = x\eta_{v\delta}/\sqrt{(Ra_{x,v})}$ and consequently $d\delta_v/dx = \delta_v/2x$.

NUMERICAL SOLUTIONS

There are three parameters in the transformed problem, namely, R, Sc and Sh. The first two parameters arise from the liquid layer equations through the suction term in the boundary condition, while the last parameter arises from the energy balance equation across the interface. We now proceed to obtain the numerical solutions to the problem using a procedure similar to that of Sparrow and Cess [5] who solved the equations for the vapor layer and the liquid layer separately with prescribed values of $\eta_{v\delta}$ and $f_L(0)$.

(1) For a prescribed value of $\eta_{v\delta}$, solution for the vapor layer can be computed according to equations (27)–(30). The results for $\theta'_v(0)$ and $\theta'_v(\eta_{v\delta})$ vs $\eta_{v\delta}$ are plotted in Fig. 2.

(2) For a prescribed value of $f_L(0)$ numerical integration of equations (32)–(37) were carried out using the Runge–Kutta method. Results for $\theta'_L(0)$ vs $f_L(0)$ are plotted in Fig. 3.

(3) For prescribed values of R, Sc and $\eta_{v\delta}$, the values of $f_L(0)$ and $\theta'_v(\eta_{v\delta})$ can be determined according to equation (35) and Fig. 2, respectively. With the value of $f_L(0)$ thus obtained, the value of $\theta'_L(0)$ is determined from Fig. 3. Finally, the value of Sh can be determined from equation (39). Results of $\eta_{v\delta}$ vs Sh at different values of Sc and R are presented in Fig. 4. Note that the

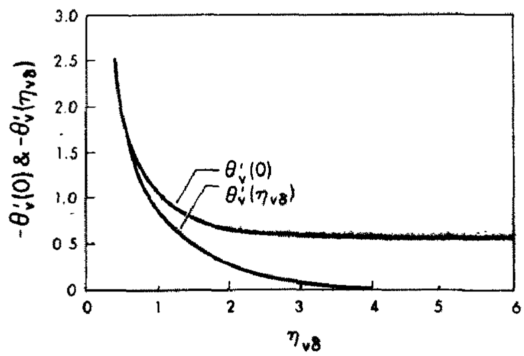

FIG. 2. Dimensionless temperature gradients in the vapor phase vs $\eta_{v\delta}$.

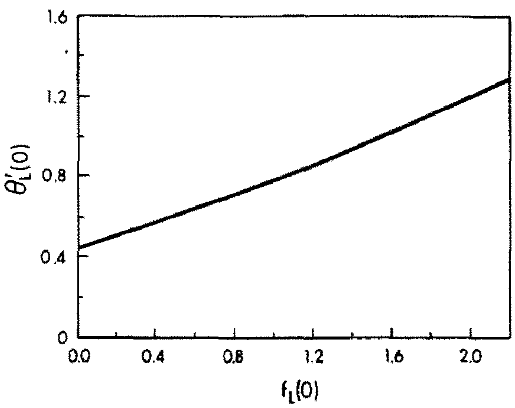

FIG. 3. $\theta_L'(0)$ vs $f_L(0)$ in the liquid phase.

uppermost curves are for the case of zero subcooling (i.e. $T_\infty = T_s$), for which the value of $\eta_{v\delta}$ depends only on Sh and independent of R, which has been discussed in the previous work by Cheng [7]. Figure 4 also shows that the effect of subcooling on $\eta_{v\delta}$ is larger at smaller R.

RESULTS AND DISCUSSION

Heat transfer results

The local surface heat flux is given by

$$q_w = -k_{m,v}\left(\frac{\partial T_v}{\partial y}\right)_{y=0} \qquad (40a)$$

which can be expressed in terms of the similarity variables to give

$$q_w = \frac{k_{m,v}(T_w - T_s)\sqrt{Ra_{x,v}}}{x}[-\theta_v'(0)]. \qquad (40b)$$

To examine the effects of liquid subcooling and wall superheating on surface heat flux, it is convenient to rewrite equation (40b) as

$$\frac{C_{pv} q_w x}{k_{m,v} h_{fg}\sqrt{Ra_{x,v}}} = Sh[-\theta_v'(0)] \qquad (40c)$$

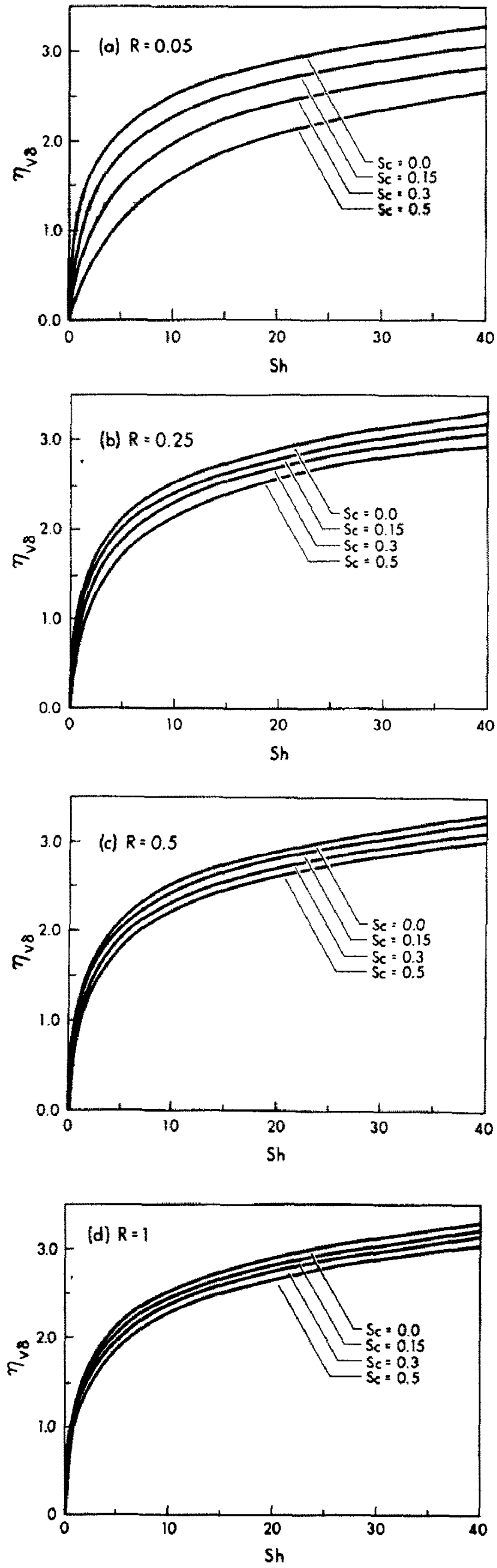

FIG. 4. $\eta_{v\delta}$ vs Sh and Sc at (a) $R = 0.05$; (b) $R = 0.25$; (c) $R = 0.5$; (d) $R = 1$.

where the right-hand side of equation (40c) is plotted vs Sh in Fig. 5 for the cases of (a) $Sc = 0$ and (b) $Sc = 0.05$ and $R = 0.5$, where the first case is independent of R. Figure 5 shows that the dimensionless surface heat flux increases as Sh or Sc is increased.

The local heat transfer coefficient and the local Nusselt number are defined as

$$h \equiv \frac{q_w}{(T_w - T_s)} \quad \text{and} \quad Nu_x \equiv \frac{hx}{k_{m,v}}. \quad (41a,b)$$

Substituting equation (40b) into equation (41a) yields the following expression for the local Nusselt number

$$\frac{Nu_x}{\sqrt{Ra_{x,v}}} = -\theta_v'(0). \quad (42)$$

When equation (30) is substituted into equation (42), one obtains

$$\frac{Nu_x}{\sqrt{Ra_{x,v}}} = \frac{1}{\sqrt{\pi}\,\mathrm{erf}\,(\eta_{v\delta}/2)}. \quad (43)$$

For a given value of $Ra_{x,v}$, equation (43) shows that the local Nusselt number depends uniquely on $\eta_{v\delta}$; and that the value of Nu_x increases as $\eta_{v\delta}$ increases. As shown in Fig. 4, $\eta_{v\delta}$ is a function of Sh, Sc and R. It follows that $Nu_x/\sqrt{Ra_{x,v}}$ (Fig. 6) is also a function of Sh, Sc and R. It is noted from these figures that (a) the values of $Nu_x/\sqrt{Ra_{x,v}}$ decreases as Sh or R is increased and as Sc is decreased, (b) the value of $Nu_x/\sqrt{Ra_{x,v}}$ is independent of R for zero subcooling [7], (c) the value of $Nu_x/\sqrt{Ra_{x,v}}$ approaches a value of 0.5642 asymptotically as $Sh \to \infty$. The last observation can be shown analytically if we examine equations (42) and (43) for the following asymptotic cases:

(a) $\eta_{v\delta} \to 0$. According to Fig. 4 this limiting case corresponds to the case of large subcooling and small wall superheating. For this situation, most of the heat supplied by the wall is used to heat the subcooled liquid, and only a small portion of heat supplied is available for vaporization. For $\eta_{v\delta}/2 < 1$, equations (28) and (30) can be expanded into a power series to give

$$\theta_v = 1 - \frac{\eta_v}{\eta_{v\delta}}[1 + (\eta_{v\delta}^2 - \eta_v^2)/12 + \cdots]. \quad (44a)$$

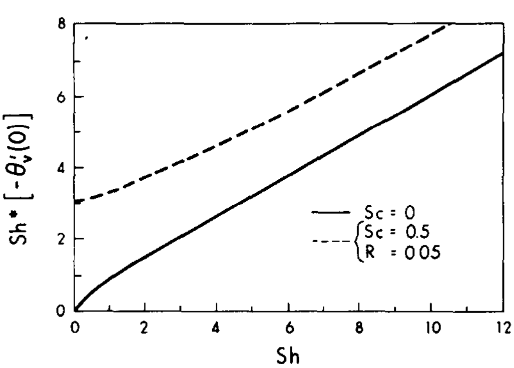

FIG. 5. Effects of Sh and Sc on the dimensionless surface heat flux.

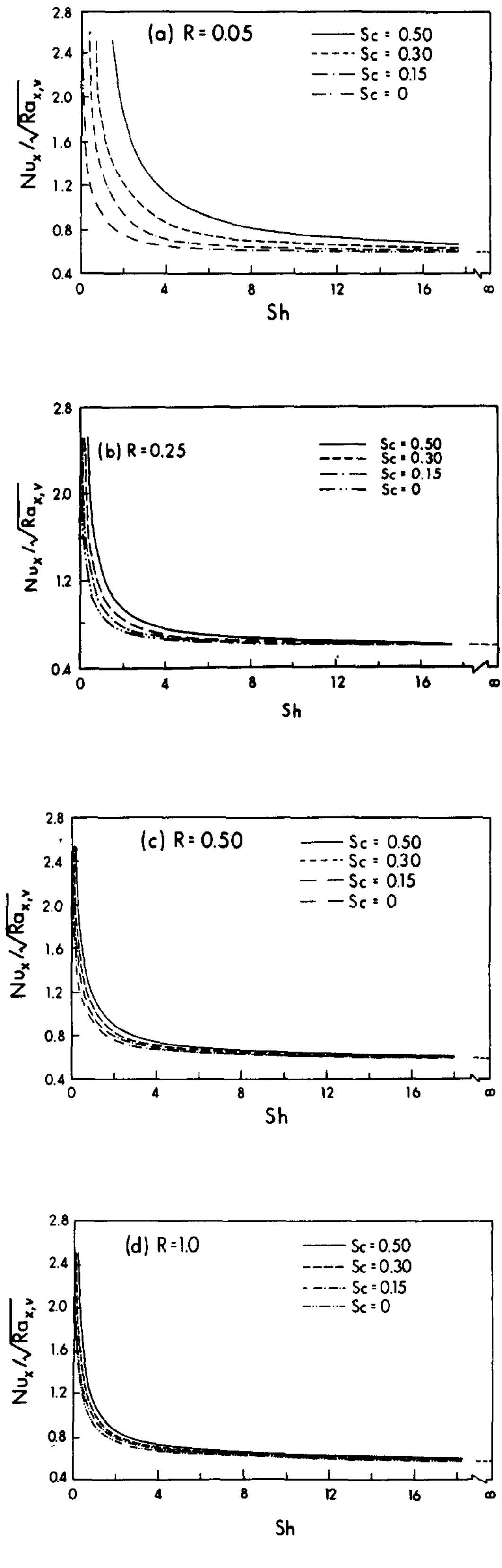

FIG. 6. (a) $Nu_x/\sqrt{Ra_{x,v}}$ vs Sh and Sc at $R = 0.05$; (b) $Nu_x/\sqrt{Ra_{x,v}}$ vs Sh and Sc at $R = 0.25$; (c) $Nu_x/\sqrt{Ra_{x,v}}$ vs Sh and Sc at $R = 0.5$; (d) $Nu_x/\sqrt{Ra_{x,v}}$ vs Sh and Sc at $R = 1$.

and

$$\theta_v' = -\frac{1}{\exp(\eta_v^2/4)\eta_{v\delta}[1-(\eta_{v\delta}/12)+\cdots]}$$

$$= -\frac{1}{\eta_{v\delta}}[1+0.0833\eta_{v\delta}^2+\cdots]. \quad (44b)$$

It follows from equations (42) and (44b) that

$$\frac{Nu_x\eta_{v\delta}}{\sqrt{Ra_{x,v}}} = 1+0.0833\eta_{v\delta}^2+\cdots, \quad \text{for } \eta_{v\delta}<2. \quad (44c)$$

Neglecting the second-order terms in equation (44) one obtains

$$\theta_v = 1-\frac{\eta_v}{\eta_{v\delta}}, \quad \text{for } \eta_{v\delta}\to 0 \quad (45a)$$

$$\frac{Nu_x\eta_{v\delta}}{\sqrt{Ra_{x,v}}} = 1, \quad \text{for } \eta_{v\delta}\to 0. \quad (45b)$$

Equation (45a) shows that η_v is a linear function of $\eta_{v\delta}$ as $\eta_{v\delta}\to 0$. Moreover, as $\eta_{v\delta}\to 0$, equation (39) gives

$$-\theta_L^{*\prime}(0) = \frac{RSh}{Sc^{3/2}\eta_{v\delta}} \quad (46)$$

where $\theta_L^{*\prime}(0)$ is the temperature gradient of the liquid phase at the interface as $\eta_{v\delta}\to 0$. Note that boundary condition (35) gives $f_L(0)\to 0$ as $\eta_{v\delta}\to 0$. Thus, $\theta_L^{*\prime}(0)$ corresponds to the temperature gradient at the wall for the case of free convection about a vertical plate without suction. Solving for $\eta_{v\delta}$ from equation (46) and substituting it into equation (45b) gives

$$\frac{hx/k_{m,L}}{[K\rho_\infty\beta_L g(T_s-T_\infty)x/\mu_L\alpha_L]^{1/2}} = -\theta_L^{*\prime}(0) \quad (47)$$

which is the same expression for Nusselt number for single-phase free convection of liquid about a vertical plate in a porous medium [9].

(b) $\eta_{v\delta}\to\infty$. According to Fig. 4, this limit corresponds to the case of large wall superheating and small subcooling. For $\eta_{v\delta}/2>1$, equations (28) and (30) become

$$\theta_v = \left[1-\text{erf}\left(\frac{\eta_v}{2}\right)\right]\left[1+\frac{\exp(-\frac{1}{4}\eta_{v\delta}^2)}{\eta_{v\delta}}+\cdots\right] \quad (48a)$$

and

$$\theta_v'(0) = -\frac{1}{\sqrt{\pi}}\left[1+\frac{\exp(-\frac{1}{4}\eta_{v\delta}^2)}{\eta_{v\delta}}+\cdots\right], \quad \text{for } \eta_{v\delta}>2. \quad (48b)$$

Substituting equation (48b) into (42) yields

$$\frac{Nu_x}{\sqrt{Ra_{x,v}}} = 0.5642\left[1+\frac{\exp(-\frac{1}{4}\eta_{v\delta}^2)}{\eta_{v\delta}}+\cdots\right], \quad \text{for } \eta_{v\delta}>2. \quad (49)$$

In the limit of $\eta_{v\delta}\to\infty$, equations (48a) and (49) give

$$\theta_v = 1-\text{erf}\left(\frac{\eta_v}{2}\right), \quad (50)$$

$$\frac{Nu_x}{\sqrt{Ra_{x,v}}} = 0.5642 \quad (51)$$

which are independent of R and Sc. Physically, equations (50) and (51) show that as $\eta_{v\delta}\to\infty$, the degree of subcooling of the surrounding fluid liquid does not affect the heat transfer characteristics in the vapor layer. Equation (51) also represents the limiting case of free convection of a dry steam, and is indicated as horizontal dashed lines at the right-hand margin of Fig. 6.

Equations (45b) and (51) suggest that a plot of $Nu_x\eta_{v\delta}/\sqrt{Ra_{x,v}}$ vs $\eta_{v\delta}$ would show clearly the limiting cases of $\eta_{v\delta}\to 0$ and $\eta_{v\delta}\to\infty$. To this end, we multiply equation (43) by $\eta_{v\delta}$ to give

$$\frac{Nu_x\eta_{v\delta}}{\sqrt{Ra_{x,v}}} = \frac{\eta_{v\delta}}{\sqrt{\pi}\,\text{erf}(\eta_{v\delta}/2)}. \quad (52)$$

The right-hand side of equation (52) vs $\eta_{v\delta}$ is plotted as a solid line in Fig. 7. The straight dashed lines in Fig. 7 represent the limiting cases of $\eta_{v\delta}\to 0$ and $\eta_{v\delta}\to\infty$ given by equations (45b) and (51), respectively. It is convenient to represent the right-hand side of equation (52) approximately by a power function of $\eta_{v\delta}$. This can be achieved by writing

$$\frac{Nu_x\eta_{v\delta}}{\sqrt{Ra_{x,v}}} = \left[1+\left(\frac{\eta_{v\delta}}{\sqrt{\pi}}\right)^m\right]^{1/m} \quad (53)$$

where the value of m is determined according to the procedures recommended by Churchill and Usagi [11] by comparing the right-hand side of equation (52) to the right-hand side of (53). As a result, we found that $m=3$, so that equation (53a) becomes

$$\frac{Nu_x\eta_{v\delta}}{\sqrt{Ra_{x,v}}} = \left[1+\left(\frac{\eta_{v\delta}}{\sqrt{\pi}}\right)^3\right]^{1/3} \quad (54)$$

which is within $\pm 5\%$ deviations from equation (52) for all values of $\eta_{v\delta}$.

Temperature and velocity profiles

The dimensionless temperature in the vapor phase, θ_v, for $\eta_{v\delta}=0.2$ to $\eta_{v\delta}=2.4$ is shown in Fig. 8 where it is noted that for $\eta_{v\delta}<0.4$, $\theta_v(\eta_v)$ varies linearly with respect to η_v. The dimensionless temperature profiles in the liquid phase, θ_L, is shown to be dependent on the value of $f_L(0)$ which in turn depends on the values of R, Sc and $\eta_{v\delta}$ as given by equation (35). It is noted that the liquid boundary layer thickness decreases with increase in Sh or decrease in Sc. The dimensionless vertical velocities f_v' and f_L' vs dimensionless distance are shown in the same plot. Note that the vertical velocity in the vapor phase is a constant which is represented by a horizontal line. As indicated by equation (32), the dimensionless vertical velocity and the dimensionless temperature profiles in the liquid phase are identical in shape, and therefore no separate representation of the vertical velocity profiles in the liquid phase is needed.

Application to heat transfer from a dike

Consider a dike 100 m in height with an average surface temperature of 400°C is intruded into an aquifer (with $K = 10^{-12}$ m^2, $k_{m,L} = 2.65$ J s^{-1} K^{-1}, and $k_{m,L} = 1.6$ J s^{-1} K^{-1}) at a temperature of 20°C. Suppose that the mean static pressure along the dike is at 10 atm and the saturated temperature corresponding to this mean pressure is 180°C. To apply the constant-property theory to this problem, we shall evaluate the properties of the vapor and liquid layers at their mean temperatures. Thus, the density, viscosity and specific heat of vapor will be evaluated at the mean temperature of $(T_w + T_s)/2 = 290$°C while that of the liquid phase at $(T_s + T_\infty)/2 = 100$°C. At these temperatures, we obtain the following properties from Hendricks, Peller and Baron [10]: $\rho_v \equiv 0.004$ g cm^{-3}, $c_{pv} = 2.16$ J g^{-1} K^{-1}, $\mu_v = 1.96 \times 10^{-4}$ g cm^{-1} s^{-1}, $c_{pL} = 4.22$ J g^{-1} K^{-1}, $\mu_L = 2.74 \times 10^{-3}$ g cm^{-1} s^{-1}, $\rho_\infty = 0.9574$ g cm^{-3} and $\beta_{L\infty} = 4.67 \times 10^{-4}$ °C^{-1}, $h_{fg} = 2019$ J g^{-1}.

With these values, we obtain $Sh = 0.2364$, $Sc = 0.3323$, and $R = 0.565$, and consequently we can determine $\eta_{v\delta}$ by interpolation from Figs. 4(c) and (d) to get $\eta_{v\delta} = 0.49$. With this value of $\eta_{v\delta}$ we obtain $Nu_x = 33.3$ at $x = 100$ m where $Ra_{x,v} = 258$ and $Ra_{x,L} = 425$. The vapor film boundary layer thickness can be determined from the definition of $\eta_{v\delta}$ which gives $\delta_v = 0.49x/\sqrt{Ra_{x,v}}$ and the vapor boundary layer thickness is given by $\delta_L = 5x/\sqrt{Ra_{x,L}}$. This is plotted in Fig. 9 where it is shown that $\delta_v = 3$ m and $\delta_L = 24.3$ m at $x = 100$ m. The vertical velocity profiles for the vapor and the liquid phases at $x = 100$ m are plotted in Fig. 10. It is shown that there is a velocity discontinuity at the vapor–liquid interface which is a consequence of the Darcy's law. The vertical velocity in the vapor phase is shown to be much higher than that in the

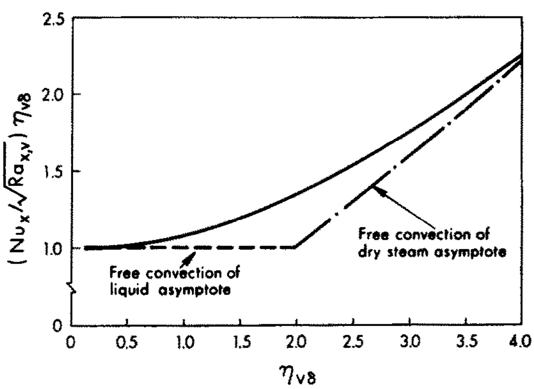

FIG. 7. $(Nu_x/\sqrt{Ra_{x,v}})\eta_{v\delta}$ vs $\eta_{v\delta}$.

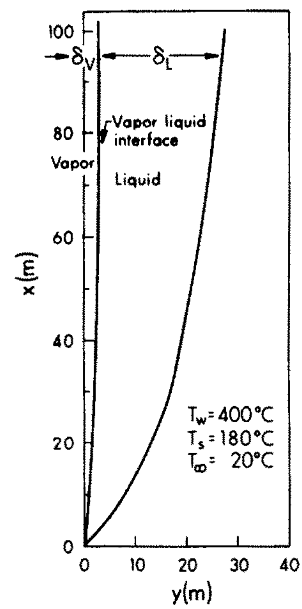

FIG. 9. Boundary layer thickness along a dike.

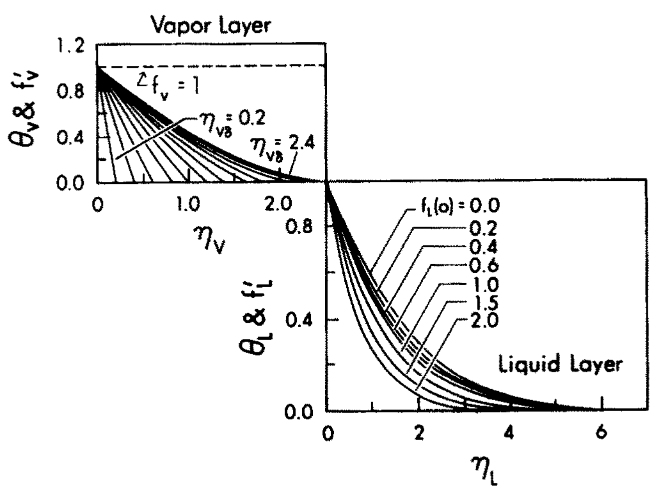

FIG. 8. Dimensionless temperature and velocity profiles in the vapor and liquid layers.

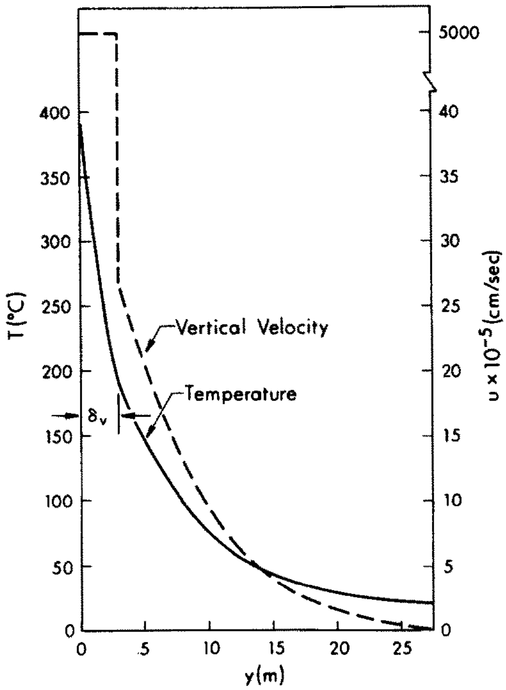

FIG. 10. Temperature and vertical velocity profiles around a dike at $x = 100$ m.

liquid phase because buoyancy force is larger and viscosity is lower for vapor.

CONCLUDING REMARKS

The problem of stable film boiling about a vertical plate in a porous medium filled with a subcooled liquid has been solved based on standard approximations in the classical film boiling literature. The validity of the present theory depends critically on the assumptions of (a) the non-existence of a two-phase zone in the boundary layer, and (b) the vapor–liquid interface being stable and smooth. For the classical film boiling problems, the first assumption appears to be widely accepted while the second assumption is more difficult to be met in reality since bubbles near the interface may be formed (resulting in a wavy shape) or detached (resulting in an unsteady behavior). As noted by Cheng [7] the first approximation is also akin to the 'abrupt interface' approximation [12] used in the investigations of seawater intrusion in freshwater aquifers, which is known to be accurate if the mixing zone is small. The applicability of these assumptions for film boiling in a porous medium can only be determined by further experiments.

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EFFET DU SOUS-REFROIDISSEMENT DU LIQUIDE SUR L'EBULLITION EN FILM AUTOUR D'UNE SURFACE VERTICALE CHAUDE DANS UN MILIEU POREUX

Résumé—On considère l'ébullition en film avec convection naturelle autour d'une plaque verticale isotherme dans un milieu poreux empli d'un liquide sous-refroidi. Par des approximations de couche limite, des solutions de similitude sont obtenues pour l'écoulement induit de la vapeur et dans les couches liquides avec un interface distinct. A un nombre de Rayleigh donné pour la vapeur, le nombre de Nusselt dépend uniquement de l'épaisseur du film de vapeur qui dépend elle-même de paramètres tridimensionnels liés au degré de surchauffe de la paroi, à l'extension du sous-refroidissement dans le liquide, aux rapports de propriétés des phases liquide et vapeur. On trouve que l'effet de l'accroissement du sous-refroidissement du fluide tend à diminuer l'épaisseur de la couche limite de vapeur, à accroître l'épaisseur de la couche limite du liquide et à augmenter le flux surfacique de chaleur. On discute l'application au transfert thermique par ébullition autour d'un dike introduit dans un aquifer.

DER EINFLUSS UNTERKÜHLTER FLÜSSIGKEIT AUF DAS FILMSIEDEN UM EINE VERTIKALE HEIZFLÄCHE IN EINEM PORÖSEN MEDIUM

Zusammenfassung—Behandelt wird das Problem des Filmsiedens bei stationärer freier Konvektion um eine beheizte isotherme vertikale Platte in einem porösen Medium, das mit unterkühlter Flüssigkeit gefüllt ist. Mit den Grenzschichtnäherungen werden Ähnlichkeitslösungen für die Auftriebsströmungen in den Dampf- und unterkühlten Flüssigkeitsschichten mit ausgeprägter Grenzfläche erhalten. Es ergibt sich, daß bei gegebener Rayleigh-Zahl des Dampfes die Nusselt-Zahl nur von der dimensionslosen Dampf-Filmdicke abhängt, die ihrerseits von drei dimensionslosen Parametern abhängig ist, die mit der Überhitzung der Wand, der Unterkühlung der umgebenden Flüssigkeit und einem Stoffwertverhältnis der Dampf- und Flüssigphase in Beziehung stehen. Es ergibt sich, daß zunehmende Unterkühlung der umgebenden Flüssigkeit zu einer Abnahme der Dicke der Dampf-Grenzschicht, zu einer Zunahme der Dicke der Flüssigkeits-Grenzschicht und zu einer Zunahme der Wärmestromdichte an der Oberfläche führt. Andererseits führt eine Zunahme der Wandüberhitzung zu einer Zunahme der Dampfschichtdicke, einer Abnahme der Flüssigkeitsschichtdicke und zu einer Zunahme der Wärmestromdichte an der Oberfläche. Die Anwendung auf den Wärmeübergang beim Sieden um heißes, in einen Aquifer eingedrungenes Ganggestein wird diskutiert.

ВЛИЯНИЕ НЕДОГРЕВА ЖИДКОСТИ НА ПЛЕНОЧНОЕ КИПЕНИЕ НА ВЕРТИКАЛЬНОЙ ПОВЕРХНОСТИ В ПОРИСТОЙ СРЕДЕ

Аннотация — Рассматривается проблема стационарного естественно-конвективного пленочного кипения на нагреваемой изотермической вертикальной пластине в пористой среде, заполненной недогретой жидкостью. С помощью приближений пограничного слоя получены уравнения подобия для описания вызванного силами выталкивания течения в слоях пара и недогретой жидкости, между которыми имеется четкая граница раздела. Найдено, что при заданном значении числа Релея для пара число Нуссельта однозначно зависит от безразмерной толщины пленки пара, которая в свою очередь зависит от трех безразмерных параметров, связанных со степенью перегрева стенки, степенью недогрева окружающей жидкости и отношением между свойствами паровой и жидкой фаз. Найдено, что с увеличением недогрева окружающей жидкости толщина пограничного слоя пара уменьшается, а толщина пограничного слоя жидкости и тепловой поток к поверхности увеличиваются. С другой стороны, увеличение перегрева стенки приводит к увеличению толщины слоя пара, уменьшению толщины слоя жидкости и увеличению теплового потока к поверхности. Рассмотрен пример переноса тепла при кипении у перемычки, помещенной в водоносном слое.